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Long-wave oscillatory Marangoni instability in a horizontal liquid layer☆

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Abstract

The long-wave instability in the problem of thermocapillary convection in a horizontal layer with a free deformable boundary and a solid bottom is investigated. The transcendental equation for the main asymptotic term of the spectral parameter is written in explicit form. The main attention is paid to investigating oscillatory instability. For the frequency of neutral oscillations, simple transcendental equations are obtained that contain the Prandtl and Biot numbers. In a number of cases, exact solutions are indicated. Explicit formulae are given for the main asymptotic term of the Marangoni number. In the case of a non-heat-conducting solid wall, the relation between the critical values of the parameters for inverse Prandtl numbers is found. It is shown that, for different Prandtl numbers, the asymptotic values are in good agreement with the numerical values.

In an investigation of thermocapillary convection in a plane horizontal liquid layer it was established[1] that the deformability of the free boundary may be the reason for instability under long-wave perturbations. Monotonic Marangoni instability was studied in Refs 2–5. Oscillatory instability was found numerically for the first time in Ref. 6, where calculations showed that it occurs only for negative Marangoni numbers, i.e. during heating of the layer from above. The thermocapillary instability in a semi-infinite layer was investigated in Ref. 7. The effect of high-frequency vibration on long-wave monotonic instability was examined in Ref. 8. A review of research on thermocapillary convection can be found in Ref. 9.

In the present paper the long-wave asymptotic forms of thermocapillary instability in a horizontal liquid layer with a free boundary and a solid wall, which may either be an isothermal or a non-heat-conducting wall, are investigated. The form of the asymptotics for the Marangoni number and for the frequency of neutral oscillations was suggested by the results of calculations[8,10] (see also: Shleikel' A L. The influence of vibration on the onset of convection in a horizontal liquid layer. Candidate Dissertation, 01.02.05, Rostov-on-Don, 2004).

1. Fundamental equations

In the study of the thermocapillary convection of a homogeneous liquid in a horizontal layer with a free deformable surface and a solid bottom, assuming that surface tension forces with a coefficient $\sigma = \sigma_0 - \sigma_T(T - T_0)$ are acting on the free surface, for dimensionless amplitudes of normal perturbations of the vertical component of velocity $\upsilon(z)$,

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temperature $\theta(z)$ and an elevation of the free boundary δ, the following spectral problem arises[3,6,8]

$$\begin{aligned}
&\lambda(D^2-\alpha^2)\upsilon = (D^2-\alpha^2)^2\upsilon, \quad \lambda\mathrm{Pr}\theta = (D^2-\alpha^2)\theta-\upsilon \\
&z = 0{:}\ \upsilon = \lambda\mathrm{Pr}\delta, \quad D^2\upsilon+\alpha^2\upsilon = \mathrm{Ma}\alpha^2(\theta+\delta) \\
&\mathrm{Cr}[(3\alpha^2+\lambda)D\upsilon-D^3\upsilon] = \alpha^2(\alpha^2+\mathrm{Bo})\delta, \quad D\theta-\mathrm{Bi}(\theta+\delta) = 0 \\
&z = 1{:}\ \upsilon = D\upsilon = 0, \quad D\theta+B\theta = 0
\end{aligned} \tag{1.1}$$

Here α is the wavenumber and $\lambda = \lambda_r + ic$ is the spectral parameter: if, for all eigenvalues $\lambda_r < 0$, stability occurs, and if at least value of λ exists, for which $\lambda_r > 0$, then instability occurs; loss of stability corresponds to the case where $\lambda_r = 0$: monotonic loss occurs if $c = 0$, and vibrational loss if $c \neq 0$; the z axis is directed downwards perpendicular to the layer, $D = d/dz$. The dimensionless parameters are given by the formulae

$$\mathrm{Pr} = \frac{\nu}{\chi}, \quad \mathrm{Ma} = \frac{A\sigma_T h^2}{\rho_0\chi\nu}, \quad \mathrm{Cr} = \frac{\rho_0\chi\nu}{\sigma_0 h}, \quad \mathrm{Bo} = \frac{\rho_0 g h^2}{\sigma_0} \tag{1.2}$$

where Pr, Ma and Bo are the Prandtl, Marangoni and Bond numbers, and Cr is the capillary parameter. Furthermore, problem (1.1) contains heat transfer parameters – the Biot numbers Bi and B. The dimensionless parameters (1.2) contain physical characteristics: the coefficient of kinematic viscosity ν, the thermal diffusivity χ, the density ρ_0 at a fixed temperature T_0, the average thickness of the layer h, the equilibrium temperature gradient A and the acceleration due to gravity g. It is assumed that $\mathrm{Cr} > 0$, and the Bond number can be positive, equal to zero ($g = 0$) or negative (an inverted layer). Subsequently, we assume that $B = B_0/B_1$, so that when $B_1 = 0$ we have an isothermal solid wall, and when $B_0 = 0$ we have a non-heat-conducting solid wall.

2. Long-wave asymptotic forms

First we will eliminate the function $\upsilon(z)$ in relations (1.1), and as $\alpha \to 0$ the unknown parameters and the function $\theta(z)$ will be sought in the form

$$\begin{aligned}
&\lambda = \lambda_0+\lambda_1\alpha^2+\ldots, \quad \mathrm{Ma} = \mathrm{Ma}_0\alpha^{-2}+\mathrm{Ma}_1+\ldots \\
&\theta(z) = \theta_0(z)+\theta_1(z)\alpha^2+\ldots, \quad \delta = \delta_0+\delta_1\alpha^2+\ldots
\end{aligned} \tag{2.1}$$

For the main terms λ_0, Ma_0, $\theta_0(z)$ and δ_0 we obtain the problem

$$\begin{aligned}
&D^6\theta_0-\lambda_0(\mathrm{Pr}+1)D^4\theta_0+\lambda_0^2\mathrm{Pr}D^2\theta_0 = 0 \\
&z = 0{:}\ D^2\theta_0-\lambda_0\mathrm{Pr}(\theta_0+\delta_0) = 0, \quad D^4\theta_0-\lambda_0\mathrm{Pr}D^2\theta_0 = \mathrm{Ma}_0(\theta_0+\delta_0)
\end{aligned} \tag{2.2}$$

$$\begin{aligned}
&\lambda_0(\mathrm{Pr}+1)D^3\theta_0-\lambda_0^2\mathrm{Pr}D\theta_0-D^5\theta_0 = 0, \quad D\theta_0-\mathrm{Bi}(\theta_0+\delta_0) = 0 \\
&z = 1{:}\ D^2\theta_0-\lambda_0\mathrm{Pr}\theta_0 = 0, \quad D^3\theta_0-\lambda_0\mathrm{Pr}D\theta_0 = 0, \quad B_1D\theta_0+B_0\theta_0 = 0
\end{aligned} \tag{2.3}$$

Below, we will derive transcendental equations for λ_0 and formulae for the critical values of Ma_0 and the frequency c in the case where $\lambda_0 = ic$.

First with we will examine the case where $\mathrm{Pr} \neq 1$. The solution of Eq. (2.2) will be sought in the form

$$\begin{aligned}
&\theta_0(z) = A_0+A_1z+A_2\mathrm{ch}\,p_1z+A_3\mathrm{sh}\,p_1z+A_4\mathrm{ch}\,p_2z+A_5\mathrm{sh}\,p_2z \\
&p_1 = \sqrt{\mathrm{Pr}\lambda_0}, \quad p_2 = \sqrt{\lambda_0}
\end{aligned}$$

Substituting $\theta_0(z)$into the boundary conditions, we obtain a homogeneous system of linear algebraic equations in the unknowns $\delta_0, A_0, \ldots, A_5$. First of all, we will write equations corresponding to the boundary conditions with $z = 0$.

We have

$$p_2^2A_4 - \lambda_0\Pr(A_0 + A_4 + \delta_0) = 0$$
$$p_1^4A_2 + p_2^4A_4 - \lambda_0\Pr(p_1^2A_2 + p_2^2A_4) = \mathrm{Ma}_0(A_0 + A_2 + A_4 + \delta_0)$$
$$\lambda_0(\Pr + 1)(p_1^3A_3 + p_2^3A_5) - \lambda_0^2\Pr(A_1 + p_1A_3 + p_2A_5) - (p_1^5A_3 + p_2^5A_5) = 0$$
$$A_1 + p_1A_3 + p_2A_5 - \mathrm{Bi}(A_0 + A_2 + A_4 + \delta_0) = 0$$

From the first two equations of this system we obtain the relations

$$A_4 = KA_2, \quad K = \frac{\mathrm{Ma}_0\Pr}{\lambda_0^2\Pr(1-\Pr) - \mathrm{Ma}_0} \tag{2.4}$$

From the other two equations it follows that $A_1 = 0$, and the following equality holds

$$p_1A_3 + p_2A_5 = \mathrm{Bi}(1 + \Pr^{-1}K)A_2 \tag{2.5}$$

From boundary conditions (2.3) we derive a further three equations

$$\begin{aligned}
&A_0 + A_4\operatorname{ch}p_2 + A_5\operatorname{sh}p_2 = \Pr^{-1}(A_4\operatorname{ch}p_2 + A_5\operatorname{sh}p_2)\\
&B_1[p_1(A_2\operatorname{sh}p_1 + A_3\operatorname{ch}p_1) + p_2(A_4\operatorname{sh}p_2 + A_5\operatorname{ch}p_2)] +\\
&+ B_0[A_0 + (A_2\operatorname{ch}p_1 + A_3\sin p_1) + (A_4\operatorname{ch}p_2 + A_5\operatorname{sh}p_2)] = 0\\
&A_4\operatorname{sh}p_2 + A_5\operatorname{ch}p_2 = 0
\end{aligned} \tag{2.6}$$

From the last equation of system (2.6) and equality (2.5) we obtain

$$A_5 = -\operatorname{cth}p_2A_2, \quad A_3 = \frac{p_2}{p_1}\operatorname{cth}p_2A_4 + \frac{\mathrm{Bi}(\Pr^{-1}K + 1)}{p_1}A_2$$

Substituting these expressions into the second equation of system (2.6), we derive a further expression for A_4 in terms of A_2

$$A_4 = \frac{\operatorname{sh}p_2[p_1f_2 + \mathrm{Bi}(1 + \Pr^{-1}K)f_1]}{p_2[p_1B_1 - \operatorname{ch}p_2f_1]}A_2 \tag{2.7}$$

where

$$f_1 = B_1p_1\operatorname{ch}p_1 + B_0\operatorname{sh}p_1, \quad f_2 = B_1p_1\operatorname{sh}p_1 + B_0\operatorname{ch}p_1 \tag{2.8}$$

Now, from Eq. (2.7) we obtain another expression for the coefficient K:

$$K_1 = -\frac{\operatorname{ch}p_2(p_1f_2 + \mathrm{Bi}f_1)}{\Pr^{-1}B_0p_1 + (p_2\operatorname{sh}p_2 + \mathrm{Bi}\Pr^{-1}\operatorname{ch}p_2)f_1} \tag{2.9}$$

Further, two problems can be examined. One of these consists of finding, for fixed values of the parameters Pr, Ma_0, Bi, B_1 and B_0, the unknown complex parameter λ_0 from the transcendental equation obtained by making the expressions for K, according to the second formula of system (2.4), equal to expression (2.9). Another problem consists of investigating the oscillatory instability, for which it is necessary to assume that $\lambda_0 = ic$ and to find c and the critical values of Ma_0. This problem will be examined below. The transcendental equation Im $K_1 = 0$ for the frequency c is obtained from the condition for the coefficient K to be real, which follows from the second formula of system (2.4). Solving this equation, we find c, then the parameter $K = K_1$, and finally we obtain

$$\mathrm{Ma}_0 = \frac{c^2\Pr(\Pr - 1)}{1 + \Pr K^{-1}} \tag{2.10}$$

As can be seen, the main terms of the asymptotic form depend only on the Prandtl and Biot numbers.

3. The case of a zero biot number

Below, we will assume that Bi = 0, and we will confine ourselves to considering a non-heat-conducting ($B_1 = 1$, $B_0 = 0$) or isothermal ($B_1 = 0$, $B_0 = 1$) solid wall.

In the first case, the equation Im $K_1 = 0$ takes the form

$$\mathrm{Im}\left(\frac{\mathrm{th}\, p_1}{\mathrm{th}\, p_2}\right) = 0 \tag{3.1}$$

In the second case

$$\mathrm{Im}\left(\frac{\mathrm{ch}\, p_1 \mathrm{ch}\, p_2}{1 + \sqrt{\mathrm{Pr}}\,\mathrm{sh}\, p_1 \,\mathrm{sh}\, p_2}\right) = 0 \tag{3.2}$$

We will introduce the notation $P = \sqrt{Pr}$ and $c = x^2/2$. Then, $p_1 = Px(1 + i)/2$ and $p_2 = x(1 \pm i)/2$.

Calculating the imaginary parts, we reduce Eq. (3.1) to the form

$$\mathrm{sh}\, Px \sin x - \sin Px\, \mathrm{sh}\, x = 0 \tag{3.3}$$

For the coefficient K we have the expression

$$K = -\frac{P}{4}\frac{\mathrm{sh}\, Px\, \mathrm{sh}\, x + \sin Px \sin x}{\left(\mathrm{ch}^2\frac{x}{2} - \cos^2\frac{x}{2}\right)\left(\mathrm{sh}^2\frac{Px}{2} + \cos^2\frac{Px}{2}\right)} \tag{3.4}$$

In the second case, for the unknown x, Eq. (3.2) can be written in the form

$$\mathrm{sh}\frac{(P+1)x}{2}\sin\frac{(P+1)x}{2} + \mathrm{sh}\frac{(P-1)x}{2}\sin\frac{(P-1)x}{2} = \frac{P}{2}(\mathrm{sh}\, Px \sin x + \mathrm{sh}\, x \sin Px) \tag{3.5}$$

In this case

$$K = -4P^2\left(\mathrm{sh}^2\frac{x}{2} + \cos^2\frac{x}{2}\right)\left(\mathrm{sh}^2\frac{Px}{2} + \cos^2\frac{Px}{2}\right)\left\{P(\mathrm{sh}\, Px\, \mathrm{sh}\, x - \sin Px \sin x) + \right.$$
$$\left. + 2\left[\mathrm{ch}\frac{(P+1)x}{2}\cos\frac{(P+1)x}{2} + \mathrm{ch}\frac{(P-1)x}{2}\cos\frac{(P-1)x}{2}\right]\right\}^{-1} \tag{3.6}$$

Eqs. (3.3) and (3.5) turned out to be fairly simple to solve numerically. In some cases they can be investigated analytically and precise solutions can be indicated.

3.1. A non-heat-conducting solid wall

We will use Eq. (3.3). For its roots, the relation

$$x(1/P) = Px(P) \tag{3.7}$$

is satisfied. This means that it is sufficient, for example, to find roots when $P > 1$. Furthermore, from this it follows that, for low wavenumbers α, the frequency c of neutral oscillations when $P < 1$ is greater than when $P > 1$, since

$$c(1/P) = P^2 c(P) \tag{3.8}$$

It is not difficult to prove that, if $P > 1$ is an integer, then Eq. (3.3) only has the roots $x = n\pi$ ($n = 1, 2, \ldots$). Then $c_n = n^2\pi^2/2$, and for the coefficient K we obtain the expression

$$K_n = -\frac{P}{4}\frac{\mathrm{sh}\, Pn\pi\, \mathrm{sh}\, n\pi}{\left(\mathrm{ch}^2\frac{n\pi}{2} - \cos^2\frac{n\pi}{2}\right)\left(\mathrm{sh}^2\frac{Pn\pi}{2} + \cos^2\frac{Pn\pi}{2}\right)} \tag{3.9}$$

Substituting c and K_n into formula (2.10), we find the corresponding number Ma_0. Depending on the eveness parity of the numbers n and P, we have:when $n=2k+1$

$$\mathrm{Ma}_{0,n} = \frac{n^4\pi^4\mathrm{Pr}(\mathrm{Pr}-1)}{4\left(1-\sqrt{\mathrm{Pr}}\,\mathrm{cth}\frac{n\pi}{2}B_n\right)}, \quad B_n = \begin{cases} \mathrm{th}\frac{\sqrt{\mathrm{Pr}}n\pi}{2}, & \sqrt{\mathrm{Pr}} = 2l+1 \\ \mathrm{cth}\frac{\sqrt{\mathrm{Pr}}n\pi}{2}, & \sqrt{\mathrm{Pr}} = 2l \end{cases} \tag{3.10}$$

when $n=2k$

$$\mathrm{Ma}_{0,2k} = \frac{4\pi^4k^4\mathrm{Pr}(\mathrm{Pr}-1)}{1-\sqrt{\mathrm{Pr}}\,\mathrm{th}k\pi\,\mathrm{cth}(\sqrt{\mathrm{Pr}}k\pi)} \tag{3.11}$$

Assuming $n=1$, from formula (3.10) we obtain $\mathrm{Ma}_{0.1}=-24\,344$ when $\mathrm{Pr}=100$, and $\mathrm{Ma}_{0.1}=-3282$ when $\mathrm{Pr}=25$.

We will now consider the case when $\mathrm{Pr}=Q^{-2}<1$, so that $\mathrm{Pr}=Q^{-1}$ and $Q>1$ is an integer. Then, as was established above, $x_n=Qn\pi$ and $c=Q^2n^2\pi^2/2$; for the coefficient K we obtain

$$K_n = -\frac{1}{4Q}\frac{\mathrm{sh}n\pi\,\mathrm{sh}Qn\pi}{\left(\mathrm{ch}^2\frac{Qn\pi}{2}-\cos^2\frac{Qn\pi}{2}\right)\left(\mathrm{sh}^2\frac{n\pi}{2}+\cos^2\frac{n\pi}{2}\right)} \tag{3.12}$$

and for the Marangoni numbers we have:when $n=2k+1$

$$\mathrm{Ma}_{0,n} = \frac{n^4\pi^4(1-\mathrm{Pr}^{-1})}{4\left(1-\mathrm{Pr}^{1/2}\mathrm{th}\frac{n\pi}{2}C_n\right)}, \quad C_n = \begin{cases} \mathrm{cth}\frac{\mathrm{Pr}^{-1/2}n\pi}{2}, & \mathrm{Pr}^{-1/2} = 2l+1 \\ \mathrm{th}\frac{\mathrm{Pr}^{-1/2}n\pi}{2}, & \mathrm{Pr}^{-1/2} = 2l \end{cases} \tag{3.13}$$

when $n=2k$

$$\mathrm{Ma}_{0,n} = \frac{n^4\pi^4(1-\mathrm{Pr}^{-1})}{4\left(1-\mathrm{Pr}^{1/2}\mathrm{cth}\frac{n\pi}{2}\mathrm{th}\frac{\mathrm{Pr}^{-1/2}n\pi}{2}\right)} \tag{3.14}$$

For example, assuming $n=1$ and $\mathrm{Pr}=0.01$, from formula (3.13) we obtain $\mathrm{Ma}_{0.1}=-2654$.

3.2. An isothermal solid wall

We now return to Eq. (3.5). It can be proved that, for uneven values of the parameter P, this equation only has the roots $x=\pi, 2\pi, \ldots$. For the proof we must put $P=2k+1$ and reduce Eq. (3.5) to the form

$$g_k(x) \equiv \frac{2(\mathrm{sh}(k+1)x\sin(k+1)x+\mathrm{sh}kx\sin kx)}{(2k+1)\mathrm{sh}(2k+1)x\sin x} - \frac{\mathrm{sh}x\sin(2k+1)x}{\mathrm{sh}(2k+1)x\sin x} = 1$$

Now it is sufficient to show that, for all $k=1, 2, \ldots$, the function $g_k(x)<1$ when $x\in(0,\infty)$, $x\neq\pi, 2\pi \ldots$; we have omitted the proof in view of its length.

Note that, if P is an even number, then Eq. (3.5) has the roots

$$x_{2l} = 2l\pi, \quad x_{2l+1} \approx (2l+1)\pi, \quad l = 1, 2, \ldots$$

If $P=Q^{-1}$ and Q is an integer, then calculations show that property (3.7) is satisfied, but not for the first root, and when $P<1$ it must be found numerically.

4. The case wither Pr = 1

The solution of Eq. (2.2) has the form

$$\theta_0(z) = A_0+A_1z+(A_2+A_3z)\mathrm{ch}pz+(A_4+A_5z)\mathrm{sh}pz, \quad p = \sqrt{\lambda_0} \tag{4.1}$$

Satisfying boundary conditions (2.3), we arrive at a transcendental equation for finding the parameter p for fixed Ma_0, Bi, B_0 and B_1:

$$K(\mathrm{th}\,p + 2\mathrm{Bi}\,p^{-1}) + \mathrm{Bi}\,p + \frac{B_0(p^2\mathrm{ch}^2 p + 2K) + p^2 B_1(K + p\,\mathrm{sh}\,p\,\mathrm{ch}\,p)}{(B_0\mathrm{sh}\,p + pB_1\mathrm{ch}\,p)\mathrm{ch}\,p} = 0$$

$$K = \frac{p^2\mathrm{Ma}_0}{2(p^4 - \mathrm{Ma}_0)} \tag{4.2}$$

If we are interested in the critical values of parameter Ma_0, we must put $\lambda_0 = ic$, $c > 0$. Then the coefficient K will be pure imaginary. We find it from Eq. (4.2), and from the condition Re $K = 0$ we obtain an equation for the unknown $p = \sqrt{ic}$. When Bi = 0 we have

$$K = -\frac{p^2\mathrm{ch}\,p(B_0\mathrm{ch}\,p + pB_1\mathrm{sh}\,p)}{2B_0 + p^2B_1 + \mathrm{sh}\,p(B_0\mathrm{sh}\,p + pB_1\mathrm{ch}\,p)}$$

The condition Re $K = 0$ when $p = x(1+i)/2$ and $B_1 = 0$ or $B_0 = 0$ leads respectively to the equations

$$\sin x = 0, \quad \mathrm{th}\,x = \mathrm{tg}\,x \tag{4.3}$$

The roots of the first equation $x = n\pi$ ($n = 1, 2, \ldots$), and the second equation has a denumerable set of roots, and here $x_1 = 3.926$ and $x_n = (\pi/4 + n\pi)$.

In the first case we have

$$c = \frac{n^2\pi^2}{2}, \quad \mathrm{Ma}_0 = \begin{cases} -\dfrac{n^4\pi^4}{2}\mathrm{th}^2\dfrac{n\pi}{2}, & n = 2k+1, \quad k = 0, 1, \ldots \\ -\dfrac{n^4\pi^4}{2}\mathrm{cth}^2\dfrac{n\pi}{2}, & n = 2k, \quad k = 1, 2, \ldots \end{cases}$$

In the second case

$$c \approx \frac{\pi^2(1+4n)^2}{32}, \quad \mathrm{Ma}_0 = -\frac{x^4}{2}\frac{x\,\mathrm{sh}\,2x + \mathrm{ch}^2 x - \cos^2 x}{\mathrm{ch}^2 x - \cos^2 x - 2x^2}$$

Note that, in all the cases considered, the results of calculations[8,10] (see also: Shleikel' A L. The influence of vibration on the emergence of convection in a horizontal layer of fluid. Candidate Dissertation, 01.02.05, Rostov-on-Don, 2004) differ from the asymptotic values by no more than 0.5%.

5. The relations between the critical values of parameters with inverse Prandtl numbers

If the solid wall is a non-heat-conducting wall, then the following assertion holds. Suppose, for fixed Pr and Bi numbers, that the main terms of the asymptotic form are $\lambda_0 = ic$ and Ma_0; then, for $\mathrm{Qr} = \mathrm{Pr}^{-1}$ and $\tilde{\mathrm{Bi}} = \mathrm{QrBi}$, the corresponding parameters are as follows:

$$\tilde\lambda_0 = \mathrm{Qr}^{-1}\lambda_0, \quad \tilde{\mathrm{Ma}}_0 = \mathrm{Qr}[\mathrm{Ma}_0 + \tilde\lambda_0^2(1 - \mathrm{Qr})] \tag{5.1}$$

The proof is provided by the fact that, with the replacement indicated, we have the following relation between the corresponding coefficients K and $\tilde K$ (see formulae (2.4) and (2.9))

$$\tilde K(\mathrm{Pr}^{-1}, \tilde{\mathrm{Bi}}, \tilde\lambda_0, \tilde{\mathrm{Ma}}_0) = K^{-1}(\mathrm{Pr}, \mathrm{Bi}, \lambda_0, \mathrm{Ma}_0) \tag{5.2}$$

Then, if Im $K = 0$, we have Im $\tilde K = 0$. In particular, when Bi = 0, the first formula of system (5.1) was obtained directly from the equations, while the second formula is verified by substitution.

In conclusion, we note that the asymptotic form given above will be the same under the action of vibration, since, in the corresponding eigenvalue problem, the vibration terms contain the factor α^2.[8,10] This is also confirmed by calculations

(Refs 8,10, etc.). The same asymptotic form occurs in the case of a free non-deforming wall (Zen'kovskaya S Long-wave oscillatory instability of thermocapillary flows in a horizontal layer. VINITI Dep. No. 1135-V2005, 9.08.2005, Rostov-on-Don, 2005).

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